UDC 541.18

A NOTE ON TWO-PHASE FILM MASS TRANSFER WITHOUT MASS FORCES

L. P. Kholpanov

Inzhenerno-Fizicheskii Zhurnal, Vol. 8, No. 5, pp. 579-585, 1965

Some aspects of two-phase film mass transfer in the absence of mass forces are examined. The results are novel in that the parametric equations contain expressions which take into account the physical properties of the contact phases.

Quite a few papers [1-10] have been devoted to the theory of film adsorption. A number of interesting conclusions have been drawn and applied with success to the analysis of all kinds of processes in two-phase film mass and heat transfer.

Film mass transfer in the absence of mass forces is described by a system of partial differential equations: the equation of motion of the liquid film and the gas and the equation of convective diffusion of the dispersoid.

For $y < \delta$ the motion of a liquid film δ on a horizontal plane is described by the Prandtl boundary layer equation

$$u_{x(l)}\,\partial u_{x(l)}/\partial x + v_{y(l)}\partial u_{x(l)}/\partial y = \nu_l\, \partial^2 u_{x(l)}/\partial y^2, \tag{1}$$

$$\partial u_{x(l)}/\partial x + \partial v_{y(l)}/\partial y = 0. \tag{2}$$

For $y > \delta$ the motion of the gas is described by the same Prandtl equation

$$u_{x(g)}\,\partial u_{x(g)}/\partial x + v_{y(g)}\,\partial u_{x(g)}/\partial y = \nu_g \partial^2 u_{x(g)}/\partial y^2, \tag{3}$$

$$\partial u_{x(g)}/\partial x + \partial v_{y(g)}/\partial y = 0. \tag{4}$$

The equation of convective diffusion is

$$u_{x(l)}\,\partial c/\partial x + v_{y(l)}\,\partial c/\partial y = D_l\,\partial^2 c/\partial y^2. \tag{5}$$

The boundary conditions are

$$y = 0,\ u_{x(l)} = v_{y(l)} = 0,\ c = c_1; \tag{6}$$

$$y = \delta,\ u_{x(l)} = u_{x(g)},\ c = c_0; \tag{7}$$

$$y = \delta,\ \mu_l\,\partial v_{y(l)}/\partial y = \mu_g \partial v_{y(g)}/\partial y,\ y \to \infty,\ u = u_\infty. \tag{8}$$

It is required to solve equations (1)-(4) together with boundary conditions (6)-(8).

Let us introduce the stream functions for the liquid and the gas:

$$\psi_l = f(\eta)\sqrt{\nu_g u_\infty x},\ \psi_g = G(\eta)\sqrt{\nu_g u_\infty x}; \tag{9}$$

$$\eta = y\sqrt{u_\infty/\nu_g x}. \tag{10}$$

In accordance with (9) and (10), the velocity components take the form

$$u_{x(l)} = \partial\psi_l/\partial y,\ v_{y(l)} = -\partial\psi_l/\partial x,\ u_{x(g)} = \partial\psi_g/\partial y,\ v_{y(g)} = -\partial\psi_g/\partial x. \tag{11}$$

In terms of the new variables, Eqs. (1)-(8) become

$$f''' + (\nu_g/2\nu_l)ff'' = 0, \tag{12}$$

$$G''' + (1/2)GG'' = 0, \tag{13}$$

$$\eta = 0,\ f(0) = f'(0) = 0, \tag{14}$$

$$\eta = \eta_\delta,\ f'(\eta_\delta) = G'(\eta_\delta),\ \mu_l f''(\eta_\delta) = \mu_g G''(\eta_\delta), \tag{15}$$

$$\eta \to \infty,\ G'(\infty) = 1. \tag{16}$$

The solution of (13) with conditions (15) and (16) has the form

$$G(\eta) \sim F(\eta - \gamma), \tag{17}$$

where γ is a constant which must be determined with the help of (15) and (16)

It is known that a solution of (12) and (13), taking into account (17), may be sought in series form

$$f(\eta) = \nu_l/\nu_g[\alpha\eta^2/2 - \alpha^2\eta^5/2\cdot 5! + 11\alpha^3\eta^8/2^2\cdot 8! - \ldots],$$
$$G(\eta) = F_w + (F''_w/2)(\eta - \gamma)^2 - (F_w F''_w/2\cdot 3!)(\eta - \gamma)^3 + \ldots$$

At comparatively small distances from the wall these equations take the form

$$f(\eta) = (\nu_l \alpha/\nu_g 2)\eta^2,\quad G(\eta) = F_w + (F''_w/2)(\eta - \gamma)^2. \tag{18}$$

The constants α, γ are found from boundary conditions (14), (15), and (16):

$$\alpha = (\nu_g/\nu_l)^2(\rho_g/\rho_l)F''_w,\quad \gamma = \eta_\delta(1 - \mu_g/\mu_l). \tag{19}$$

Since $\mu_l \gg \mu_g$, $\gamma = \eta_\delta$.

Values of F''_w at small distances from the wall are equal to 0.332 [11].

Knowing the relation $f(\eta)$ and hence the velocity distribution in the liquid film, we may solve the convective transfer equation (5) for film mass transfer.

Let

$$u_{x(l)} = u_\infty f'(\eta),\quad v_{y(l)} = (1/2)(u_\infty \nu_g/x)^{1/2}[\eta f'(\eta) - f(\eta)]. \tag{20}$$

With these values of the velocity components Eq. (5) may be satisfied if the concentration is a function of η alone. In fact, if we put

$$c = c_1 - (c_1 - c_0)\theta(\eta),$$

then Eq. (5) becomes

$$d^2\theta/d\eta^2 + (\nu_g/2D_l)f(\eta)\,d\theta/d\eta = 0. \tag{21}$$

The boundary conditions are:

$$\eta = 0,\quad \theta(\eta) = 0,\quad \eta \to \eta_\delta,\quad \theta(\eta) = 1. \tag{22}$$

Then Eq. (21) takes the form

$$\theta(\eta) = \beta(\mathrm{Pr})\int_0^\eta \exp\left[-(\nu_g/2D_l)\int_0^t f(\xi)\,d\xi\right]dt. \tag{23}$$

Let us evaluate $\beta(\mathrm{Pr})$ approximately. We substitute values of $f(\xi)$, α, and η_δ given respectively by (10), (20), and (22).

We obtain

$$\beta(\mathrm{Pr}) = \int_0^k \exp(-\omega^3)\,d\omega,$$

where

$$k \sim 5.2\,[(F''_w/12)\,\mathrm{Pr}_l\,(\mu_g/\mu_l)^2(\gamma_l/\gamma_g)]^{1/2}.$$

Let us evaluate the upper limit of the integral as applied to a film scrubber for absorption of sulfur dioxide from annealing furnace gases using water:

$$\gamma_g = 1.16\ \mathrm{kg/m^3},\ \gamma_l = 1000\ \mathrm{kg/m^3},\ \mu_g = 0.02\ \mathrm{cp},$$
$$\mu_l = 1\ \mathrm{cp},\quad \mathrm{Pr}_l = 10^3 - 10^4.$$

In this case the upper limit of the integral is greater than unity. Since the integrand diminishes very rapidly for values of the argument greater than unity, it may be replaced by infinity. Direct computation of the integral $\beta(\mathrm{Pr})$ leads to the same result:

$$\int_0^u [\exp(-x)/x^{2/3}]\,dx = \gamma(1/3,\ u),$$

where

$$u = (F''_w/12)\,\mathrm{Pr}_l\,(\mu_g/\mu_l)^2(\gamma_l/\gamma_g).$$

For the above example of absorption of sulfur dioxide by water, at large values of u the incomplete gamma function (γ) differs from its values for infinity by approximately 4%, which is quite acceptable in engineering calculations.

The diffusion flux at a horizontal surface has the form

$$j = D_l\,(c_1 - c_0)\,\beta(\mathrm{Pr})\sqrt{u_\infty/\nu_g x}.$$

At large Pr numbers β(Pr), taking account of [11], has the form

$$\beta(\mathrm{Pr}) = (1/2.9)\,\mathrm{Pr}^{1/3}(\mu_g/\mu_l)^{2/3}(\rho_l/\rho_g)^{1/3}.$$

The diffusion flux at large Pr numbers is then

$$j = (D_l/2.9)(c_1 - c_0)\,\mathrm{Pr}^{1/3}(\mu_g/\mu_l)^{2/3}(\rho_l/\rho_g)^{1/3}\sqrt{u_\infty/\nu_g x}. \tag{24}$$

From (24) the diffusion layer δ is calculated thus:

$$1/\delta = \mathrm{Pr}^{1/3}(2.9)^{-1}(\mu_g/\mu_l)^{2/3}(\rho_l/\rho_g)^{1/3}\sqrt{u_\infty/\nu_g x}.$$

Taking into account that the dynamic layer on the plate

$$\delta_0 \sim \sqrt{\nu_g x/u_\infty},$$

we have

$$\delta_0 = \delta\,\mathrm{Pr}^{1/3}(\mu_g/\mu_l)^{2/3}(\rho_l/\rho_g)^{1/3}.$$

Thus the relations between the dynamic δ_0' and the diffusion δ layers of single- and two-phase flows differ by an amount $(\mu_g/\mu_l)^{2/3}(\rho_l/\rho_g)^{1/3}$.

This conclusion agrees with experimental data for mass transfer in bubbling [13].

The total flux to the plate has the form

$$I_{\mathrm{dif}} = \int j\,dx\,dz = bl^{1/2}(5.8)^{-1}D(c_1 - c_0)\,\mathrm{Pr}^{1/3}\times(\mu_g/\mu_l)^{2/3}(\rho_l/\rho_g)^{1/3}\sqrt{u_\infty/\nu_g},$$

or in criterial form

$$\mathrm{Nu} = 0.12\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}(\mu_g/\mu_l)^{2/3}(\rho_l/\rho_g)^{1/3}. \tag{25}$$

According to (20), the velocity at the phase boundary is

$$u_x = u_\infty(\mu_g/\mu_l)F''_w\,\delta_0\sqrt{u_\infty/\nu_r x}.$$

Since $\delta_0 \sim \sqrt{\nu_g x/u_\infty}$, the velocity

$$u_x = Au_\infty(\mu_g/\mu_l). \tag{26}$$

For laminar motion of the film and the transverse velocity distribution given by (26), the convective diffusion equation has the form

$$u_x\partial c/\partial x = D\partial^2 c/\partial y^2. \tag{27}$$

With the boundary conditions:

$$\text{when } y = 0\ c = c_0,\ \text{when } y \to \infty\ c = 0 \tag{28}$$

Eq. (27) has the solution:

$$c = (2c_0/\sqrt{\pi})\int_0^b \exp(-z^2)\,dz,$$

where

$$b = (y/2)\,[(Au_\infty/Dx)(\mu_g/\mu_l)]^{1/2}.$$

The formula for the diffusion flux density on the plate surface is

$$j = D(dc/dy)_{y=0} = c_0\sqrt{(ADu_\infty/\pi x)(\mu_g/\mu_l)}.$$

The effective thickness of the diffusion layer is

$$\delta = Dc_0/j = \sqrt{\pi D x \mu_l / A u_\infty \mu_g}.$$

The total flux at the surface of a liquid film of width b and length l is

$$I = c_0 b \sqrt{4 D l u_\infty A \mu_g / \pi \mu_l}.$$

The criterial equation of film mass transfer is then

$$\mathrm{Nu} = \sqrt{4A/\pi}\, \mathrm{Pe}^{1/2} (\mu_g/\mu_l)^{1/2}. \tag{29}$$

Equation (29) is novel in that it takes into account the physical properties of the contact phases. This equation coincides in form with the equation obtained experimentally and theoretically for mass transfer in bubbling [13].

Let us represent the stream functions for the gas and the liquid as

$$\psi_l = f(\eta) \sqrt{\nu_l u_l x}, \quad \psi_g = G(\eta) \sqrt{u_g \nu_g x}.$$

Then the normal and tangential velocity components become

$$u_{x(l)} = \partial \psi_l / \partial y, \quad v_{y(l)} = -\partial \psi_l / \partial x,$$
$$u_{x(g)} = \partial \psi_g / \partial y, \quad v_{y(g)} = -\partial \psi_g / \partial y.$$

We substitute these velocities in (1)-(8) to obtain

$$f''' + (1/2) f f'' = 0, \quad G''' + (1/2) G G'' = 0 \tag{30}$$

with boundary conditions

$$\begin{gathered} \eta = 0, \ f(0) = f'(0) = 0; \\ \eta = \eta_\delta, \ u_l f'(\eta_\delta) = u_g G'(\eta_\delta), \ \mu_l u_l \eta_\delta f''(\eta_\delta) = \mu_g u_g \eta_\delta G''(\eta_\delta); \\ \eta \to \infty, \quad G'(\infty) = 1. \end{gathered} \tag{31}$$

The solution may be represented in series form

$$\begin{gathered} f(\eta) = (\alpha/2)\eta^2 - (\alpha^2/2\cdot 5!)\eta^5 + \ldots \\ G(\eta) = F_w + (F''_w/2)(\eta - \gamma)^2 - (F_w F''_w/2\cdot 3!)(\eta - \gamma)^3 + \ldots \end{gathered} \tag{32}$$

At very small distances from the wall we need retain only the first terms:

$$f(\eta) = (\alpha/2)\eta^2, \quad G(\eta) = F_w + (F''_w/2)(\eta - \gamma)^2 - \ldots \tag{33}$$

Using conditions (31), we find the constant coefficient in (33)

$$\alpha = (\mu_g/\mu_l)(\nu_l/\nu_g)^{1/2} (u_g/u_l)^{3/2}. \tag{34}$$

In accordance with (34), function (33) becomes

$$f(\eta) = (\mu_g/2\mu_l)(\nu_l/\nu_g)^{1/2} (u_g/u_l)^{3/2} \eta^2. \tag{35}$$

Knowing the value of this function, we can easily find the diffusion flux. Taking into account (23), (24), and (35), we express it as

$$\begin{gathered} j = (D_l/2.9)(c_1 - c_0) \mathrm{Pr}^{1/3} (\mu_g/\mu_l)^{1/3} (\nu_l/\nu_g)^{1/6} \times \\ \times (u_g/u_l)^{1/2} \sqrt{u_l/\nu_l x}. \end{gathered}$$

After determination of the total diffusion flux, the criterial equation of mass transfer takes the form

$$\mathrm{Nu} = 0.12\, \mathrm{Re}^{1/2} \mathrm{Pr}^{1/3} (\mu_g/\mu_l)^{1/3} (\nu_l/\nu_g)^{1/6} (u_g/u_l)^{1/2}. \tag{36}$$

The factor taking account of the phase state is, in this case, the expression

$$(\mu_g/\mu_l)^{1/3} (\nu_l/\nu_g)^{1/6} (u_g/u_l)^{1/2}.$$

The velocity at the boundary of the two phases, with Eq. (35) taken into account, has the form

$$u_x = A u_l (\mu_g/\mu_l)(\nu_l/\nu_g)^{1/2} (u_g/u_l)^{3/2}.$$

Let us find the criterial equation of film mass transfer for the given stream function. This is

$$\mathrm{Nu} = \sqrt{4A/\pi}\, \mathrm{Pe}^{1/2} (\mu_g/\mu_l)^{1/2} (\nu_l/\nu_g)^{1/4} (u_g/u_l)^{3/4}. \tag{37}$$

Finally, let us write the stream function for the gas and the liquid in the following form:

$$\psi_l = f(\eta)\sqrt{u_\infty \nu_l x}, \quad \psi_g = G(\eta)\sqrt{u_\infty \nu_g x}.$$

In accordance with these values of the stream functions, the equations for $f(\eta)$ and $G(\eta)$ respectively take the form (30), which must be solved for the following boundary conditions:

$$\eta = 0, \quad f(0) = f'(0);$$
$$\eta = \eta_\delta, \; f'(\eta_\delta) = G'(\eta_\delta), \; (\mu_l/\nu_l^{1/2})\, f''(\eta_\delta) = (\mu_g/\nu_g^{1/2})\, G''(\eta_\delta); \tag{38}$$
$$\eta \to \infty, \quad G'(\infty) = 1.$$

The solution of Eqs. (30) has the same form as (32). The constant α in (32), determined from conditions (38), has the form

$$\alpha = (\mu_g/\mu_l)(\nu_l/\nu_g)^{1/2} F''_w. \tag{39}$$

Consequently, the function $f(\eta)$ for very small distances from the wall, taking account of (39), has the form

$$f(\eta) = (1/2)(\mu_g/\mu_l)(\nu_l/\nu_g)^{1/2} F''_w \eta^2. \tag{40}$$

Taking $f(\eta)$ from (40) and performing calculations for the diffusion flux similar to those performed above, we write the criterial equation for the given case as

$$\mathrm{Nu} = 0.12\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}(\mu_g/\mu_l)^{1/3}(\nu_l/\nu_g)^{1/6}. \tag{41}$$

It follows from (41) that the factor which takes account of the properties of the contact phases is $(\mu_g/\mu_l)^{1/3}(\nu_l/\nu_g)^{1/6}$.

With account for the value of function (40), the velocity at the boundary of the two phases has the form

$$u_x = A u_l (\mu_g/\mu_l)(\nu_l/\nu_g)^{1/2}. \tag{42}$$

The solution of the convective diffusion equation (27), with boundary conditions (28) and account for velocity (42), is given by the expression

$$\mathrm{Nu} = 0.12\sqrt{4A/\pi}\,\mathrm{Pe}^{1/2}(\mu_g/\mu_l)^{1/2}(\nu_l/\nu_g)^{1/4}. \tag{43}$$

In (25), (29), (36), (37), (41), and (43), the diffusion parameter Nu should be calculated as described in [12] for film absorbers.

According to (43), the factor taking account of the phase state is represented by the expression

$$(\mu_g/\mu_l)^{1/2}(\nu_l/\nu_g)^{1/4}.$$

Thus, the criterial equation describing two-phase film mass transfer must be written as

$$\mathrm{Nu} = A\,\mathrm{Pe}^{1/2} f.$$

Here, as shown above, the nature of factor f varies depending on choice of the stream function, and hence on the velocity distribution for the gas and liquid flows.

NOTATION

u_∞ – velocity of undisturbed flow; ψ_l, ψ_g – stream functions for liquid and gas, respectively; ν_l, ν_g – viscosity of liquid and gas, respectively; f – factor taking into account the physical properties of the contact phases; $\beta(\mathrm{Pr}) = 1/\int_0^{\eta_\delta} \exp[-(\nu_g/2D_l)\int_0^t f(\xi)\,d\xi]\,dt$.

REFERENCES

1. V. V. Vyazov, ZhTF, 10, 1519, 1940.
2. P. A. Semenov, ZhTF, 14, 427, 1944.
3. P. L. Kapitsa and S. P. Kapitsa, ZhETF, no. 2, 1949.
4. M. D. Kuznetsov, ZhPKh, no. 1, 1948.
5. M. E. Pozin, ZhPKh, 21, 218, 1948.
6. P. L. Kapitsa, ZhETF, 18, 3, 1948.
7. N. M. Zhavoronkov, Hydraulic Basis of Scrubber Action [in Russian], Izd-vo "Sovetskaya nauka," 1944.
8. H. Hikita, and Y. Oko, Chem. Engng (Japan), 23, 808, 1959.
9. J. Rossum, J. Chem. Sci., 2, 35, 1959.

10. G. A. Ratcliff and K. J. Ried, Trans. Inst. Chem. Engrs., 39, 423, 1961.
11. H. W. Emmons and D. C. Leigh, Aer. Res. Coun. Report, 10, 1915, 1953.
12. Yu. I. Dytnerskii, A. G. Kasatkin, and L. P. Kholpanov, ZhPKh, no. 7, 1964.
13. A. G. Kasatkin, Chemical Engineering Processes and Equipment [in Russian], Goskhimizdat, 1960.

17 June 1964

Polytechnic Institute,
Tula